

DC Motor:

Principle of operation:

A machine that converts electrical energy into mechanical energy is known as motor.

Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force.

The magnitude of the force is given by

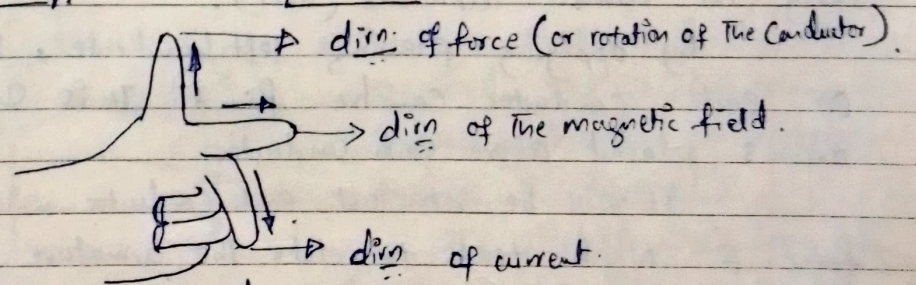
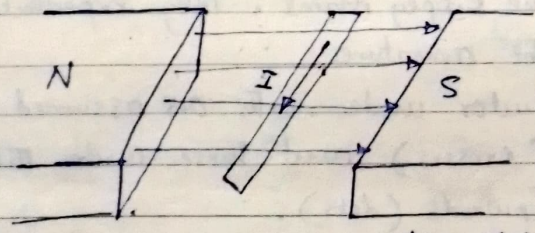
$$F = BIL \text{ newtons.}$$

- Where $B \rightarrow$ magnetic flux density wb/m^2
- $I \rightarrow$ current carried by the conductor
- $L \rightarrow$ length of the conductor lying in the magnetic field.

The dirⁿ of the force is given by Fleming's left hand rule.



Fleming's left hand rule:



Stretch the left hand such that the first 3 fingers (i.e. Thumb, fore finger & middle finger) are mutually \perp or to each other then thumb points the dirⁿ of force (or rotation of the conductor), the fore finger points dirⁿ of the magnetic field and the middle finger points the dirⁿ of current.

Apply FLH rule to this fig, it is clear that the force acting on the conductor vertically upwards.

~~force~~ ~~act~~ ~~down~~ ~~of~~ ~~conductor~~

————— A —————

How the armature rotates

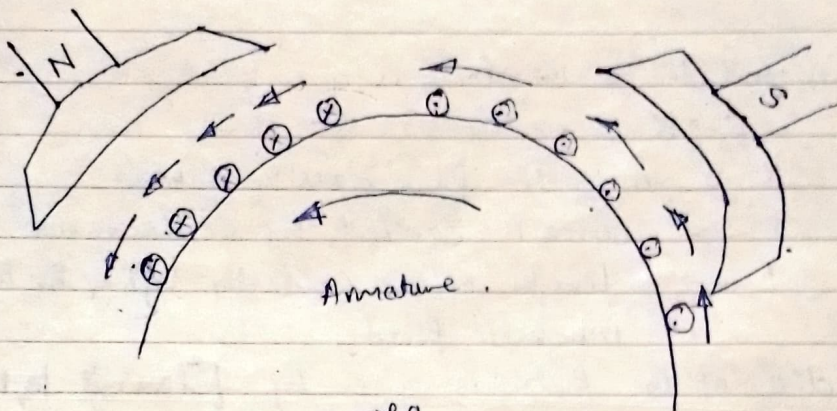


Fig Shows ^{a part of a} multipolar dc motor. When its field magnet are excited and its armature conductors are supplied with currents from the supply mains, They experiences a force tending to rotate the armature.

Armature conductor under north are assumed to carry current downwards (crosses) and those under the south pole carry the current upwards (dots).

By applying Fleming's left hand rule, the dirⁿ of force on each conductor can be found. It is shown by small arrows placed above each conductor.

It will be seen that each conductor will experiences a force F which tends to rotate the armature in the anticlockwise dirⁿ. These force collectively produces a driving torque which sets the armature rotating.

When the conductor under the south pole will carry the current upwards due to the rotation of the armature that conductor comes under influence of ~~the~~ north pole at that instant the current in that conductor reverses its dirⁿ from upwards to downward and the dirⁿ of force on the conductor remains the same.

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Back emf or counter emf:-

direction of the

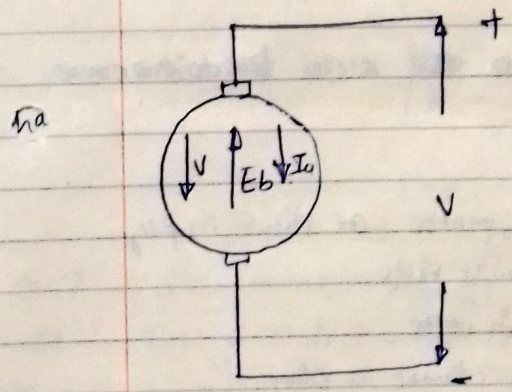
As soon as the supply is given to the dc motor, the motor starts rotating. The rotating armature cuts the magnetic flux produced by the poles, an emf will be induced in the armature conductor according to Faraday's laws of electromagnetic induction. The induced emf in the conductor acts against to the applied voltage V (in accordance with the Lenz's law) and hence it is known as back or counter emf (E_b)

Since the back emf generated in the armature, the expression for back emf is ~~same~~ same as that of expression for generated emf in a generator

$$E_b = \frac{\Phi Z N P}{60 X A} \text{ volts}$$

The back emf is always less than the applied voltage (V), although this difference is very small when the motor running under normal condition.

Significance of back emf:-



As the armature rotates back emf (E_b) opposes the applied voltage as shown in fig. The applied voltage has to force the current through the armature against the back emf.

The electric work done in overcoming the opposition is converted into mechanical energy developed in the armature. So the energy conversion in a dc motor is possible due to the production of back emf (E_b).

\therefore net voltage across the armature circuit = $V - E_b$
 $\therefore I_a = \frac{V - E_b}{R_a}$

Since V and R_a are usually (constant) fixed, the value of E_b determines the current drawn by the motor.

$$\therefore \left[E_b = \frac{72 \text{ HP}}{60 \text{ A}} \right]$$

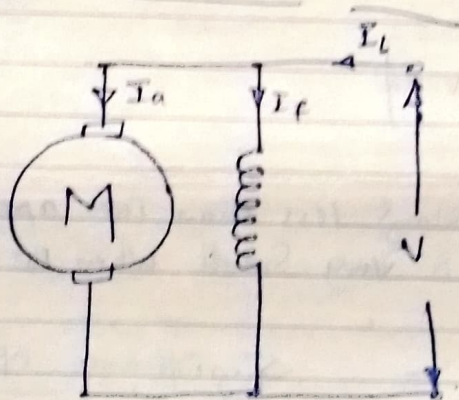
$E_b \propto \omega \phi$

If speed is high, E_b is max and the motor will draw less armature current.

If speed is low, E_b is small and the motor will draw large armature current, ~~increases~~ ~~presence~~ of ~~back emf~~ which develops more torque. Hence the presence of back emf makes the dc motor a self-regulating unit.

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Voltage equation of a motor:



~~The voltage~~ ~~applied~~ ~~to~~ ~~the~~ ~~motor~~ ~~is~~ ~~given~~ ~~by~~
~~the~~ ~~equation~~ ~~below~~

Consider a dc shunt motor as shown in fig.

let $V =$ applied voltage in volts

$E_b =$ back emf in volts

$R_a =$ armature resistance in Ohm

$I_a =$ armature current.

Since back emf (E_b) acts in opposition to applied voltage (V).

The net voltage across the armature circuit is $(V - E_b)$

The armature current I_a is given by

$$I_a = \frac{V - E_b}{R_a}$$

$$\text{Or } \boxed{V = E_b + I_a R_a} \rightarrow (1)$$

Eq. (1) is known as the voltage eqⁿ of the motor.

If the brush drop is considered then the voltage eqⁿ becomes

$$\boxed{V = E_b + I_a R_a + B.D}$$

~~It~~ ~~eqn~~ \times by I_a on both side

$$V I_a = E_b I_a + I_a^2 R_a$$

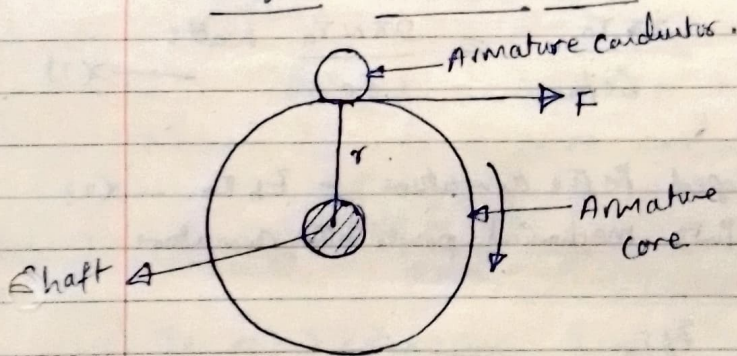
Where $V I_a \rightarrow$ electric p^r to the armature in watts

$E_b I_a \rightarrow$ power developed in the armature in watts

$I_a^2 R_a \rightarrow$ Armature or cu loss.

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Torque of a DC Motor:



Torque is the turning moment of a force about an axis and is measured by the product of force and radius of the armature at right angles to which the force acts.

$$T = F \times r \quad \text{N-m}$$

In a dc motor, each conductor is acted upon by a circumferential force F at a distance ' r '.

The radius of the armature. Hence each conductor exerts a torque, tending to rotate the armature.

The sum of the torque, due to all armature conductors is known as gross torque (or armature torque).

Expression for torque of a dc motor:-

let

$Z \rightarrow$ no. of conductors on the armature

$P \rightarrow$ no. of pole

$\Phi \rightarrow$ flux/pole in wb

$N \rightarrow$ Speed in rpm

$A \rightarrow$ no. of parallel path

$I_a \rightarrow$ armature current

$r \rightarrow$ radius of the armature in m

$T_a \rightarrow$ Armature torque N-m

Work done in one revolution of the armature

$$= \text{Force} \times \text{distance travelled.}$$

$$= F \cdot 2\pi r \quad \text{joules.}$$

$$= (F \times r) \cdot 2\pi$$

$$= 2\pi T_a$$

\therefore power developed in the armature.

$$= \frac{\text{Work done}}{\text{time taken}} \quad \text{in watts.}$$

$$= \frac{2\pi T_a}{60/N} = \frac{2\pi N T_a}{60} \quad \text{watts} \quad \text{--- (1)}$$

We also have,

$$\text{Power developed in the armature} = E_b I_a \quad \text{--- (2)}$$

The Electrical power converted into mechanical power in the armature

$$(1) = (2)$$

$$\frac{2\pi N T_a}{60} = E_b I_a \quad \text{--- (3)}$$

$$\therefore \frac{T_a \times 2\pi N}{60} = \frac{\Phi Z P}{60 A} I_a$$

$$T_a = \left(\frac{P Z}{2\pi A} \right) \Phi I_a, \quad \text{N-m}$$

$$T_a = \frac{0.159 Z P}{A} \Phi I_a \quad \text{N-m}$$

$$T_a = \left(\frac{0.159}{9.81} \right) \Phi Z I_a \left(\frac{P}{A} \right) = 0.0162 P Z I_a \left(\frac{P}{A} \right)$$

Also (1) (9)

$$T_a = \frac{E_b I_a}{\left(\frac{2\pi N}{60} \right)} = 9.55 \left(\frac{E_b I_a}{N} \right) \quad \text{N-m.}$$

Shaft torque:



The torque which is available at the shaft or some part of it is known as shaft torque.

All the torque developed in the motor is not available at the shaft or some part of it. It is lost for compensating mechanical losses.

Hence the difference of T_a and T_{loss} is available at the shaft.

Hence, $T_{sh} = T_a - T_{loss}$

Types of ~~torque~~

Output = $T_{sh} \times 2\pi N$ watts

Where T_{sh} is in N-m and N is in rpm.

$\therefore T_{sh} = \frac{\text{O/P in watts}}{2\pi N}$ N-m

If N is in rpm then

$T_{sh} = \frac{\text{O/P in watts}}{\frac{2\pi N}{60}}$ N-m

$= \left(\frac{60}{2\pi}\right) \frac{\text{O/P in watts}}{N}$ N-m

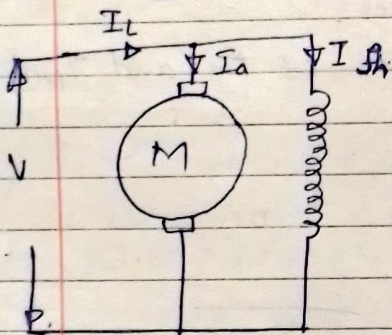
$T_{sh} = 9.55 \frac{\text{O/P in watts}}{N}$ N-m

Type of DC Motors:-

Similar to generator, The ^{dc} motor also classified into 3 types

- (1) Shunt Wound
- (2) Series Wound
- (3) Compound Wound.

(i) Shunt Wound:-

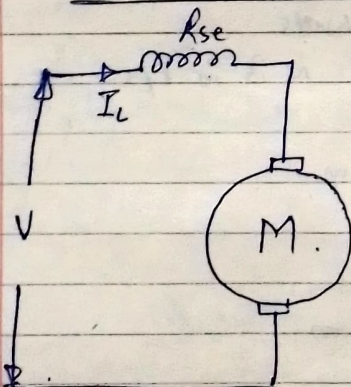


$$E_b = V - I_a R_a$$

$$I_a = I_L - I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

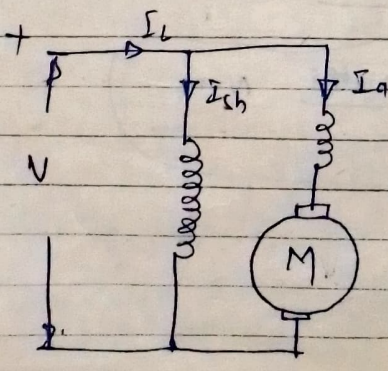
(ii) Series Wound motor:-



$$E_b = V - I_a R_a - I_a R_{se}$$

$$I_L = I_{se} = I_a$$

(iii) Compound motor:-



$$E_b = V - I_a R_a - I_a R_{se}$$

$$I_a = I_L - I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

Speed of a Dc motor:-

From the voltage equation of a motor we get

$$E_b = V - I_a R_a$$

We have $E_b = \frac{\phi Z N P}{60 A}$

$$\frac{\phi Z N P}{60 A} = V - I_a R_a$$

$$\therefore N = \left(\frac{60 A}{Z P} \right) \frac{(V - I_a R_a)}{\phi}$$

$$\therefore N = \left(\frac{E_b}{\phi} \right) \left(\frac{60 A}{Z P} \right) \text{ rpm}$$

$$\therefore N = K \left(\frac{E_b}{\phi} \right)$$

It shows that speed is directly proportional to the back emf E_b and inversely proportional to the flux.

$$\therefore \boxed{N \propto \frac{E_b}{\phi}}$$

For series motor:-

Let N_1, I_{a1}, ϕ_1 be the speed, armature current, and flux/pole in the first case.

Let N_2, I_{a2}, ϕ_2 be the speed, armature current and flux/pole in the second case.

Then using the above relation we get

$$N_1 \propto \frac{E_{b1}}{\phi_1} \quad \text{when } E_{b1} = V - I_{a1} R_a$$

$$\text{Also } N_2 \propto \frac{E_{b2}}{\phi_2} \quad \text{when } E_{b2} = V - I_{a2} R_a$$

(i)

$$\therefore \boxed{\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}} \longrightarrow (A)$$

Before saturation of the magnetic poles
 $\Phi \propto I_a$.

$$\therefore \boxed{\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{I_{a1}}{I_{a2}}}$$

(ii) For shunt motor:

In this case the same equation applies

$$\text{i.e. (A)} \Rightarrow \frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

For shunt motor flux practically remains constant
So that $\Phi_1 = \Phi_2$

$$\therefore \boxed{\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}}$$

The DC Motor characteristics:-

The performance characteristics of all types of DC motor are listed below.

(i) Torque v/s armature current characteristics:-

(T_a v/s I_a)

It is a plot b/w armature torque v/s armature current. It is also known as the electrical char.

(ii) Speed and armature current I_a
i.e. (N v/s I_a)

At

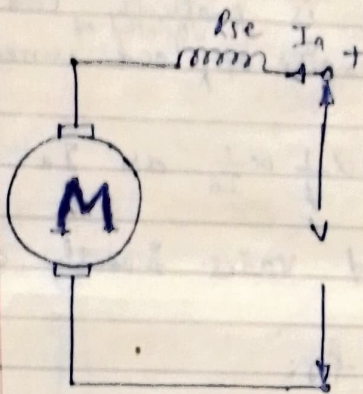
(iii). Speed vs torque characteristics.

$$N \propto 1/I_a$$

It is also known as a mechanical char.

(D) Characteristics of Series motor.

(i) Armature torque vs armature current (T_a vs I_a)
or electrical char.



For series motor,

$$T_a \propto \phi I_a$$

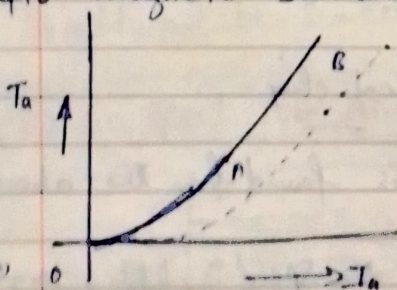
Since $\phi \propto I_a$ before saturation

$$\therefore \boxed{T_a \propto I_a^2}$$

Therefore, as I_a increases, T_a increases as square of I_a .

After saturation ϕ is constant so that
 $T_a \propto I_a$ only.

Thus upto saturation the torque is proportional to square of the armature current. Thus T_a vs I_a curve upto magnetic saturation is a parabola. i.e. curve as shown in fig.



However after magnetic saturation, the torque is directly proportional to the armature current.

$\therefore T_a$ vs I_a is almost a

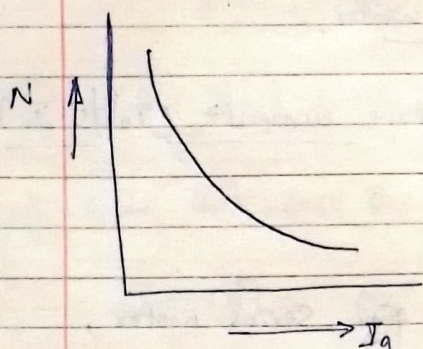
straight line (curve AB)

The shaft torque T_{sh} is less than the armature torque because of stray losses. A dotted curve depicting it in fig (c).

Hence at ~~heavy load~~, the torque of a dc series motor will be very high before magnetic saturation.

The series motor are used in the application to accelerate heavy masses quickly electric locomotives, cranes & hoists, conveyors, etc.

ie in case of \Rightarrow Speed vs Armature current char. (N vs I_a)



We have

$$N \propto \frac{E_b}{\phi}$$

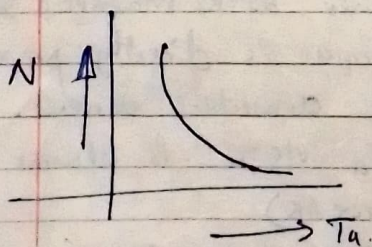
Since the drop in armature and series field is small, E_b is practically constant in spite of I_a load current.

Hence $N \propto \frac{1}{\phi} \propto \frac{1}{I_a}$ as I_a increases

ϕ also increases, hence speed varies inversely as armature current as shown in fig.

When load is heavy, I_a is large, hence speed is low. When the load is very small, I_a is small & ϕ is also small and hence speed becomes dangerously high. Hence series motor should never be started without some mechanical load on it, otherwise it may develop excessive speed and get damaged due to heavy centrifugal forces so produced.

3) N vs T_a or mechanical char



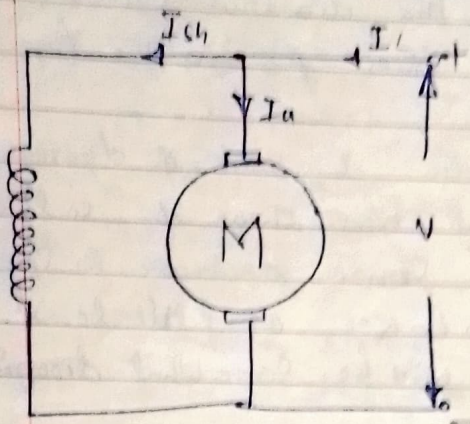
It is found from the above char

When the speed is high, torque is low and vice versa.

and the relation b/w the two as

shown in fig.

Char of Shunt motors



From fig.

$$I_L = I_a + I_f$$

$$\text{and } I_f = \frac{V}{R_f}$$

and hence the flux in a shunt motor is practically constant.

So the field current is constant.

(i) T_a vs I_a char

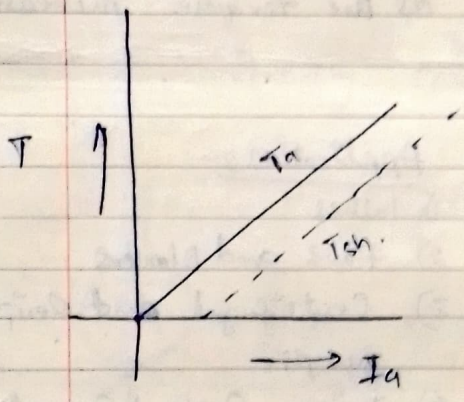
For a dc motor we have,

$$T_a \propto \phi I_a$$

Since flux is constant for shunt motor

$$T_a \propto I_a$$

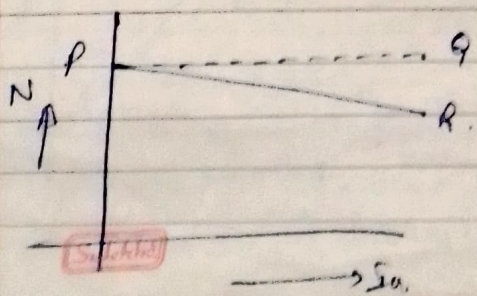
Hence T_a vs I_a char is a str. line passing through the origin as shown in fig.



and the shaft torque which is less than the armature torque and is as shown in the fig.

It is clear from the fig that a larger armature current is required to start ^{a load} Hence shunt motors should not be started on heavy load.

(ii) Speed vs Armature current char.



For dc motor we have

$$N \propto \frac{E_b}{\phi}$$

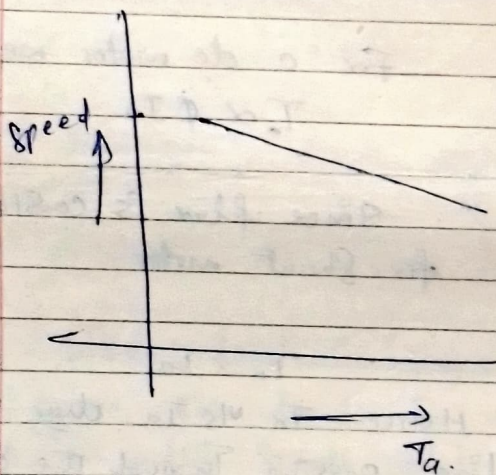
The flux ϕ and back emf E_b is almost constant in a shunt motor under normal load condition.

Therefore the speed of the shunt motor ~~is almost~~ will remain constant as the armature current varies and is shown by dotted lines ϕ in the fig.

At large loads, both E_b and ϕ decreases, but E_b decreases somewhat more than ϕ , so that all considered, there is some decrease in speed, the drop ranging from 5 to 15% of full load. Thus actual speed curve will be somewhat drooping as shown by line AC.

3) N/T_a char.

This curve is obtained by plotting N vs T_a for various armature currents.



It may be seen that speed falls somewhat as the torque increases.

Applications:-

- 1) Lathes
- 2) Fans and Blowers.
- 3) Centrifugal and Reciprocating pumps.
- 4) Driving Constant Speed Shafting.

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